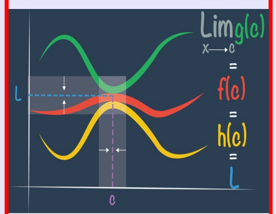
Math 261

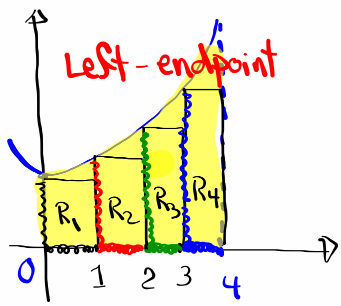
Fall 2022

Lecture 37



Feb 19-8:47 AM

Find the area below $f(x) = x^2 + 1$, above x-axis from $x=0$ to $x=4$.



$A_1 = 1 \cdot f(0)$

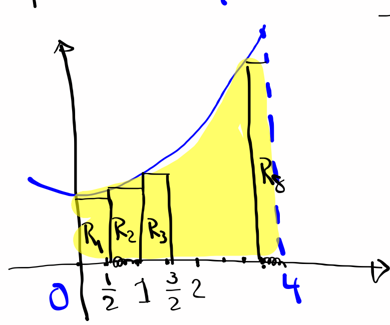
$A_2 = 1 \cdot f(1)$

$A_3 = 1 \cdot f(2)$

$A_4 = 1 \cdot f(3)$

$\approx A_1 + A_2 + A_3 + A_4$

$\sum_{i=1}^{4} A_i$



$A_1 = \frac{1}{2} \cdot f(0/2)$

$A_2 = \frac{1}{2} \cdot f(1/2)$

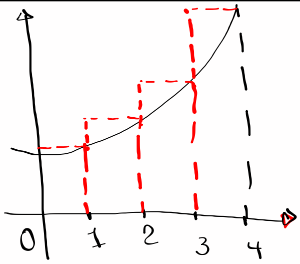
$A_3 = \frac{1}{2} \cdot f\left(\frac{2}{2}\right)$

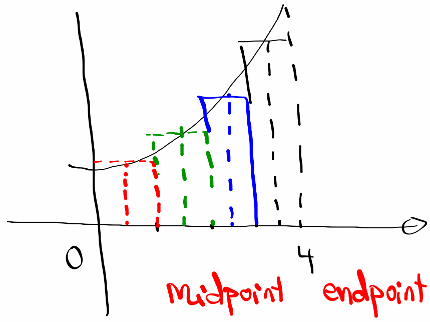
$\vdots$

$A_8 = \frac{1}{2} \cdot f\left(\frac{7}{2}\right)$

$\approx \sum_{i=1}^{8} A_i$

Nov 2-8:48 AM

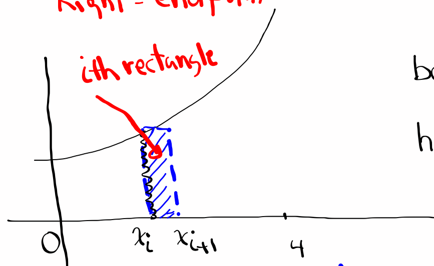




Right-endpoint

Midpoint endpoint

ith rectangle



n Subintervals

base = $x_{i+1} - x_i$

height = $f(x_i)$

$A_i = (x_{i+1} - x_i) \cdot f(x_i)$

$\sum_{i=1}^{n} A_i$

$n \to \infty$

Final answer for area $\lim_{n\to\infty} \sum_{i=1}^{n} A_i$ is limit exists.

Nov 2-9:00 AM

Consider $f(x) > 0$ for $[a,b]$

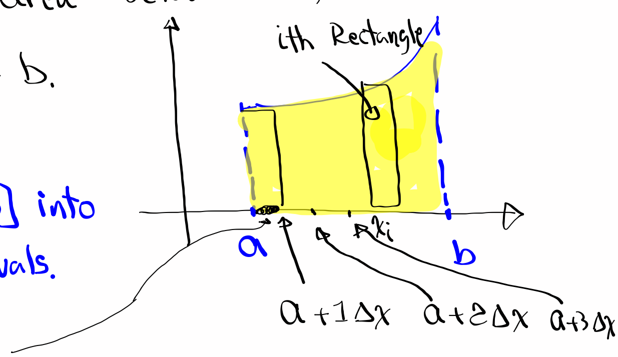
Find the area below $f(x)$, above x-axis,

for $a \le x \le b$.

Divide $[a,b]$ into n Subintervals.

Delta x

$\Delta x = \frac{b-a}{n}$



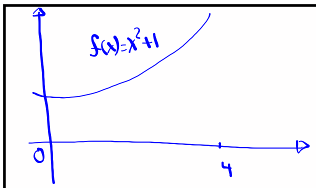
$x_i = a + i\,\Delta x$

height = $f(x_i)$

$A_i = \Delta x \cdot f(x_i)$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

Nov 2-9:11 AM



$f(x)=x^2+1$

$[a,b]=[0,4]$

$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$

$x_i = a + i\,\Delta x$

$x_i = 0 + i\cdot\frac{4}{n} = \frac{4}{n}i$

$f(x_i) = f\left(\frac{4}{n}i\right)$

$= \left(\frac{4}{n}i\right)^2 + 1$

$= \frac{16}{n^2}i^2 + 1$

$A_i = \Delta x \cdot f(x_i)$

$= \frac{4}{n}\cdot\left[\frac{16i^2}{n^2}+1\right]$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left[\frac{16i^2}{n^2}+1\right]$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(\frac{16}{n^2}i^2+1\right)$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[\sum_{i=1}^{n} \frac{16i^2}{n^2} + \sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}\cdot\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} 1\right] \qquad \sum_{i=1}^{n}1 = \underbrace{1+1+1+\cdots+1}_{n\text{ times}}$$

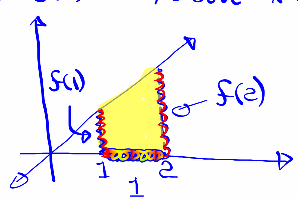
$$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} + n(1)\right] \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty}\left[\frac{128n^3+\cdots}{6n^3} + 4\right]$$

$$= \frac{128}{6} + 4 = \frac{64}{3} + 4 = \frac{64+12}{3} = \boxed{\frac{76}{3}}$$

Nov 2-9:19 AM

Find the area below $f(x)=x$, above x-axis from $x=1$ to $x=2$.



$$A_{\text{Trapezoid}} = \frac{\boxed{h}(B+b)}{2}$$

$$= \frac{1(1+2)}{2} = \boxed{\frac{3}{2}}$$

$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$

$x_i = a + i\,\Delta x$

$x_i = 1 + \frac{i}{n}$

$f(x_i) = 1 + \frac{i}{n}$

$A_i = \Delta x \cdot f(x_i)$

$= \frac{1}{n}\cdot\left[1+\frac{i}{n}\right]$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[1+\frac{i}{n}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[1+\frac{i}{n}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[n(1) + \frac{1}{n}\sum_{i=1}^{n} i\right] = \lim_{n\to\infty} \frac{1}{n}\left[n + \frac{1}{n}\cdot\frac{n(n+1)}{2}\right]$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty}\left[1 + \frac{n^2+n}{2n^2}\right] = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$$

Nov 2-9:39 AM

$$1 + 2 + 3 + 4 + \cdots + 100$$
$$100 + 99 + 98 + \cdots + 1$$

101 101 101 101

$$\frac{100(101)}{2} = \frac{100(1+100)}{2}$$

$$1 + 2 + 3 + \cdots + n$$
$$= \frac{n(1+n)}{2}$$

Carl Gauss

$$\sum_{i=1}^{n} i$$

Nov 2-9:52 AM